2.3 Three Operations

A categorical statement such as

Some employed workers are not people with health insurance.

can be rewritten as

Some people without health insurance are *not* unemployed workers.

to counter the bias that people without health insurance are people without jobs. The logic rules that enable us to do so are called the three transformation rules or operations. They are *conversion*, *contraposition* and *obversion*. When applied to the right kinds of statements, they yield new statements that are *logically equivalent* to the original statements.

Two statements are logically equivalent if they *necessarily* have the same truth value. That is, if one of them is true, the other must also be true, and if one of them is false, the other must also be false. We can illustrate that two statements $p$ and $q$ are logically equivalent in terms of the truth table by ruling out the second and the third cases.

<table>
<thead>
<tr>
<th></th>
<th>p</th>
<th>q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>

2.3.1 Conversion

The operation conversion moves the subject term to the predicate term position, and moves the predicate term to the subject term position. In short, it switches the subject and the predicate terms.
Applying an operation to a statement would yield a new statement. To see whether the new statement is logically equivalent to the original statement, we can use the Venn Diagrams. The following diagram shows how sets, their complements and the intersections are represented as different colored areas. To see the areas, move the cursor on top of each term.

![Venn Diagram](image)

Applying conversion to E and I statements will yield logically equivalent statements. From the Venn Diagrams we can see clearly why they are equivalent. Notice that the diagrams on the right-hand side are the mirror images of the ones on the left.
For example, if we apply conversion to the **E** statement

No ravens are white birds.

we would get a logically equivalent statement

No white birds are ravens.

Applying conversion to the **I** statement

Some swans are white birds.

would yield a logically equivalent statement

Some white birds are swans.

However, when applied to the **A** and **O** statements, conversion does not yield logically equivalent statements.
In every pair, the Venn diagrams are not mirror images of each other.

Applying conversion to the \textbf{A} statement

\begin{itemize}
  \item All tigers are mammals.
\end{itemize}

would result in a statement

\begin{itemize}
  \item All mammals are tigers.
\end{itemize}

which is false, and thus cannot be logically equivalent to the original statement. It is also evident that the \textbf{O} statement

\begin{itemize}
  \item Some mammals are not whales.
\end{itemize}

is not logically equivalent to

\begin{itemize}
  \item Some whales are not mammals.
\end{itemize}

\subsection{2.3.2 Contraposition}

Like conversion, the operation contraposition switches the subject and the predicate terms. Afterwards, it replaces the two terms with their complements.
Applying contraposition to A and O statements will yield logically equivalent statements. Again, this can be seen by comparing the Venn diagrams.

\[
\text{All } S \text{ are } P. = \text{ All } \text{non-}P \text{ are } \text{non-}S.
\]

\[
\begin{array}{c}
\text{Some } S \text{ are not } P. = \text{ Some } \text{non-}P \text{ are not } \text{non-}S
\end{array}
\]

Contraposition would transform the A statement

All animals that can fly are animals with wings.

into

All animals without wings are animals that cannot fly.
To see more clearly the operation at work, we can use “F” for “animals that can fly” and “W” for “animals with wings.” The term “animals that cannot fly” would then be “non-F,” and “animals without wings” would be “non-W.”

All F are W. = All non-W are non-F.

This example illustrates how contraposition helps us recognize that two sentences are logically equivalent. The pair of O statements that we saw at the beginning of this section

Some employed workers are not people with health insurance. (Some E are not H.)

and

Some people without health insurance are not unemployed workers. (Some non-H are not non-E.)

are also logically equivalent because of contraposition. So are the following two O statements

Some useful gadgets are not successful inventions. (Some U are not S.)

Some unsuccessful inventions are not useless gadgets. (Some non-S are not non-U.)

When contraposition is applied to the E and I statements, the new statements would not be logically equivalent to the original ones. We can see this clearly from their Venn Diagrams.

\[
\begin{align*}
\text{No S are P.} & \neq \text{No non-P are non-S.} \\
\begin{array}{c}
\begin{array}{c}
S \\
\end{array}
\end{array} & \neq \begin{array}{c}
\begin{array}{c}
\text{P} \\
\end{array}
\end{array}
\end{align*}
\]

\[
\begin{align*}
\text{Some S are P.} & \neq \text{Some non-P are non-S.} \\
\begin{array}{c}
\begin{array}{c}
S \\
\end{array}
\end{array} & \neq \begin{array}{c}
\begin{array}{c}
\text{P} \\
\end{array}
\end{array}
\end{align*}
\]

For example, these two E statements

No dead organisms are immortal beings. (No non-L are non-M.)
No mortal beings are living organisms. (No $M$ are $L$.)

are not logically equivalent. Nor are the following pair of $I$ statements

Some persuasive arguments are illogical. (Some $P$ are $non-L$.)

Some logical arguments are unpersuasive. (Some $L$ are $non-P$.)

2.3.3 Obversion

The operation obversion is quite different from the previous two. It involves two steps:

1. Change the quality of the statement;
2. Replace the predicate term with its complement.

Applying obversion to categorical statements will always yield logically equivalent statements.

All $S$ are $P$.

Select the statement type: $A$

Select the subject term: $S$

Select the predicate term: $P$

The first step means changing the affirmative statements to the negative statements, and the negative statements to the affirmative ones. For example, if the original statement is an $E$ statement, which is negative, obversion would transform it into an $A$ statement, which is affirmative. Afterwards, replace the predicate term with its complement.

All $S$ are $P. = No S$ are $non-P$.

No $S$ are $P. = All S$ are $non-P$. 
Some $S$ are $P$. = Some $S$ are not $\textit{non-}P$.

Some $S$ are not $P$. = Some $S$ are $\textit{non-}P$.

As we can see in the following examples, obversion is fairly common in daily language. Even though the jargon “obversion” may be new to you, you have used it countless times before.

No genetic diseases are contagious. = All genetic diseases are non-contagious.

Some medical tests are not accurate. = Some medical tests are inaccurate.

Some people are irrational. = Some people are not rational.

After an operation is applied, if the new statement is logically equivalent to the original one, then the two must have the same truth value. But if they are not logically equivalent, then the new statement does not have to share the same truth value with the original statement. This means that the truth value of the new statement may or may not be the same as the truth value of the original statement. As a result, the truth value of the new statement would be undetermined.

**Exercise 2.3**

I. Follow the instruction and practice three operations as many times as you want. You can also use it to check your answers in the next section.
Three Operations

(1) Provide a categorical statement and its truth value;
(2) Pick an operation from the drop-down menu;
(3) Derive the new statement and its truth value.

<table>
<thead>
<tr>
<th>Given Statement (Truth Value)</th>
<th>Operation</th>
<th>New Statement</th>
<th>Truth Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. All B are non-C. (T)</td>
<td>Conversion</td>
<td>________________________</td>
<td></td>
</tr>
<tr>
<td>2. Some M are not non-A. (F)</td>
<td>Contraposition</td>
<td>________________________</td>
<td></td>
</tr>
<tr>
<td>3. No H are non-G. (F)</td>
<td>Conversion</td>
<td>________________________</td>
<td></td>
</tr>
<tr>
<td>4. Some non-K are H. (T)</td>
<td>Obversion</td>
<td>________________________</td>
<td></td>
</tr>
<tr>
<td>5. No non-R are non-D. (T)</td>
<td>Contraposition</td>
<td>________________________</td>
<td></td>
</tr>
<tr>
<td>6. All non-N are W. (F)</td>
<td>Obversion</td>
<td>________________________</td>
<td></td>
</tr>
<tr>
<td>7. Some S are not non-E. (T)</td>
<td>Obversion</td>
<td>________________________</td>
<td></td>
</tr>
<tr>
<td>8. All non-P are G. (T)</td>
<td>Contraposition</td>
<td>________________________</td>
<td></td>
</tr>
<tr>
<td>9. No D are non-R. (F)</td>
<td>Contraposition</td>
<td>________________________</td>
<td></td>
</tr>
<tr>
<td>10. Some non-S are not T. (F)</td>
<td>Obversion</td>
<td>________________________</td>
<td></td>
</tr>
<tr>
<td>11. Some U are not L. (T)</td>
<td></td>
<td>Some U are non-L.</td>
<td></td>
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</tr>
<tr>
<td>12. No K are non-M. (F)</td>
<td></td>
<td>No non-M are K.</td>
<td></td>
</tr>
<tr>
<td>13. All non-B are A. (T)</td>
<td></td>
<td>No non-B are non-A.</td>
<td></td>
</tr>
<tr>
<td>14. Some F are non-D. (F)</td>
<td></td>
<td>Some D are non-F.</td>
<td></td>
</tr>
<tr>
<td>15. Some non-R are A. (F)</td>
<td></td>
<td>Some non-R are not non-A.</td>
<td></td>
</tr>
<tr>
<td>16. Some C are not non-H. (F)</td>
<td></td>
<td>Some non-H are not C.</td>
<td></td>
</tr>
<tr>
<td>17. All T are non-S. (F)</td>
<td></td>
<td>All S are non-T.</td>
<td></td>
</tr>
<tr>
<td>18. Some non-P are G. (F)</td>
<td></td>
<td>Some non-P are not non-G.</td>
<td></td>
</tr>
<tr>
<td>19. No non-D are M. (F)</td>
<td></td>
<td>No non-M are D.</td>
<td></td>
</tr>
<tr>
<td>20. All B are non-C. (T)</td>
<td></td>
<td>All non-C are B.</td>
<td></td>
</tr>
</tbody>
</table>

III. Suppose you are stranded on a desert island. In searching for foods, you discover that some bright-colored fungi on the island are edible. Use three operations and the logical relations in the Squares of Opposition to infer whether the following sentences are true (T), false (F) or undetermined (?).

1. ___ All bright-colored fungi on the island are edible.
2. ___ Some bright-colored fungi on the island are not edible.
3. ___ Some edible fungi on the island are bright-colored.
4. ___ Some edible fungi on the island are not bright-colored.
5. ___ No edible fungi on the island are bright-colored.
6. ___ All bright-colored fungi on the island are inedible.
7. ___ All edible fungi on the island are non-bright-colored.
8. ___ Some inedible fungi on the island are bright-colored.
9. ___ Some non-bright-colored fungi on the island are edible.
10. ___ All non-bright-colored fungi on the island are edible.
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